FFT-based Equalizer with Doppler Compensation for OFDM Systems in Time-Variant Multipath Channels

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Abstract—In Orthogonal Frequency Division Multiplexing (OFDM) mobile communication systems time-varying channels cause an inter-carrier interference (ICI) and degrade the performance due to the frequency orthogonality loss. Doppler shift compensation or equalization is very complex and often prohibitive in consumer receiver equipment. We propose a Doppler effect compensation / equalization technique for these kinds of scenarios. Using this technique we can compensate fast time-varying channels perfectly within an OFDM symbol with uniform Doppler shift and perfect channel knowledge. The computational complexity of the technique is the same as of FFT (N log(N)).

Index Terms—Signal processing, Mobile receiver, Doppler-shift compensation

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is one of the key techniques in recent communication systems and is based on the fast Fourier transform (FFT). Communication systems such as Long Term Evolution (LTE), Worldwide Interoperability for Microwave Access (WiMAX), Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB) etc. are based on this algorithm [1],[2]. There are many advantages of OFDM which made it preferable recently. Although the FFT computes the Discrete Fourier Transform (DFT) with a low complexity and implements OFDM efficiently, in mobile environments time-variant multipath channels destroy the orthogonality of the carriers and cause inter-carrier interference (ICI).

Mobile transmitters or receivers cause time variations due to the Doppler shifts. In linear time-varying (LTV) channels Doppler shifts destroy the orthogonality and lead to complex equalization algorithms in the receiver. In the literature LTV is also referred to as double selective channel. Recently, many compensation and equalization techniques have been proposed to reduce ICI in time-varying multipath channels, e.g. equalization [2],[6], self-cancellation [3] or minimum mean-squared error (MMSE) [4]. However, either they are very complex or not a complete solution to the problem. In addition in the best case their computational complexity is \( O(N^3) \).

In this paper we propose a new equalizer which is based on FFT calculations with a computational complexity of \( O(N \cdot \log N) \) and compensates for the channel effects including Doppler shifts in time-variant channels.

This paper is organized as follows: In section II we introduce a time-variant channel model. In section III we introduce the Doppler shift compensation technique and discuss its computational complexity and the channel estimation algorithms shortly. Section IV contains simulation results for a DVB-T2 transmission using the DVB-T2 Common Simulation Platform [12]. Finally, at the end, we give a conclusion.

II. TIME-VARIANT MULTIPATH PROPAGATION MODEL

In OFDM systems the coded symbols \( X(l), l = 0, ..., N - 1 \) enter into the IFFT block where they are processed according to (1) and the output is transmitted after parallel to serial conversion, Fig. (I). In the receiver the received symbols are converted from serial to parallel and entered into the FFT block. At the end the equalization block equalizes the channel effects.

The mathematical description of the IFFT for \( k = 0, ..., N - 1 \) is

\[
x(k) = \frac{1}{N} \sum_{l=0}^{N-1} X(l) \exp \left( j2\pi \frac{k \cdot l}{N} \right)
\]

In real wireless transmission scenarios the transmitted signal will be faded, time delayed and frequency shifted (Doppler shift). In LTV channels delays occur mostly due to multipath propagation and Doppler shifts occur due to the mobility of either the transmitter or receiver, frequency offset and oscillator drifts [5].

For a mathematical model we denote the channel transfer function with \( h(\tau,t) \), propagation paths with \( P \) and input signal with \( x(t) \). We assume that the OFDM system has a guard interval \( T_{gl} \) longer than the maximum delay in the channel. Hence we can write the received signal \( y(t) \) for \( P \) paths with attenuation \( \rho_p \), phase shift \( \phi_p \), time delay \( \tau_p \) and uniform Doppler shift \( v_p \) for \( p \)-th path as follows

\[
y(t) = \sum_{p=0}^{P-1} \rho_p e^{j\phi_p} x(t - \tau_p) e^{j2\pi v_p t} + \omega(t)
\]

\( \omega(t) \) is the noise in the channel. For clarity we will skip the noise term in the following calculations since the goal of this paper is to analyze the channel distortion caused by only LTV. By denoting \( \rho_p e^{j\phi_p} \) with \( h_p \) (2) becomes

\[
y(t) = \sum_{p=0}^{P-1} h_p x(t - \tau_p) e^{j2\pi v_p t}
\]

Equation (3) can be generalized for continuously interacting objects as follows
\[ y(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} S_H(\tau, \nu) x(t - \tau_p) e^{j2\pi\nu_p t} dt dv \quad (4) \]

\[ S_H(\tau, \nu) = \sum_{p=0}^{P-1} \rho_p e^{j\phi_p} \delta(t - \tau_p) \delta(\nu - \nu_p) \quad (5) \]

\[ \rho_p, \phi_p, \tau_p \text{ and } \nu_p \text{ can be time } t \text{ dependent. In this paper, however, we assume the coherence time of the channel to be big enough to model them as constant within one OFDM symbol. As a popular model of LTV [8] impulse response can be specified as} \]

\[ h(\tau, t) = \sum_{p=0}^{P-1} h_p \delta(t - \tau_p) e^{j2\nu_p t} \quad (6) \]

Above formulation is derived as an inverse FFT of (5) with respect to Doppler \( \nu \).

Doppler shift is the frequency change in mobile environments when at least one of the objects is moving. We denote \( \nu_p = \nu \cdot \frac{f_c}{c} \cdot \cos \phi_p \), where \( \nu \) is the speed of the object, \( \phi_p \) is the angle of arrival, \( f_c \) is the carrier frequency and \( c \) is the speed of light.

### III. Doppler shift compensation technique

As we know the FFT algorithm will convert the input signal \( y \) into the frequency domain and according to the convolution-sum representation of LTV systems \( Y(f) \) can be written as multiplication of \( X(f) \) and the Fourier transformation of the channel

\[ Y(f) = F\{y(t)\} = \sum_{p=0}^{P-1} X(f) h_p e^{-j2\pi f \nu_p} \otimes F(D_p) \quad (7) \]

Here \( D_p \) is the \( p \)-th path Doppler shift vector with samples \( k = 0, \cdots, N - 1 \) and Doppler shift \( \nu_p \)

\[ D_p(k) = e^{-j2\pi \nu_p k} \quad (8) \]

Since an addition operation in time domain is an addition in frequency domain, (7) can be written as follows

\[ Y(f) = X(f) \sum_{p=0}^{P-1} h_p e^{-j2\pi f \nu_p} \otimes F(D_p) \quad (9) \]

If we denote \( h_p e^{-j2\pi f \nu_p} \) with \( H_p(f) \), (9) will become

\[ Y(f) = X(f) \sum_{p=0}^{P-1} H_p(f) \otimes F(D_p) = X(f) H(f) \quad (10) \]

#### A. Doppler shift compensation based on FFT

Here we show the equalization process which we use for simulations. \( H(f) \) in (10) is an array and can be written as follows

\[ H(f) = \sum_{p=0}^{P-1} H_p(f) \otimes F(D_p) \quad (11) \]

We can write (11) in time domain using \( h_p \) and \( D_p \) as

\[ h(t) = \sum_{p=0}^{P-1} h_p(t) D_p(t) \quad (12) \]

Now, we can apply an FFT to this equation and get the \( H \) array. Finally, we can compensate / equalize the Doppler shifts and other effects by multiplying the received signal \( Y \) to the inverse of \( H \). This operation has linear complexity.

#### B. Doppler shift compensation in frequency domain

In section (III-A) we showed how to calculate \( H \) based on an FFT. Here we show \( H \) in the frequency domain from a matrix inversion point of view. Hence, we denote \( H_p = [H_{p0}, \cdots, H_{p(N-1)}] \) as a complex vector for path \( p \). The convolution operation in (7) can be constructed as a matrix multiplication, where one of the inputs is converted into a circulant matrix. \( H_p \) can be written in a circulant matrix form as follows

\[ H_c = \begin{pmatrix} H_{p0} & H_{p(N-1)} & \cdots & H_{p1} \\ H_{p1} & H_{p0} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ H_{p(N-1)} & \vdots & \cdots & H_{p0} \end{pmatrix} \quad (13) \]

Using matrix \( H_c \) the convolution operation can be written as follows

\[ H_c Y = X \]

[Diagram: OFDM system model]
\[
H_p(f) \otimes \mathcal{F}(D_p) = H_c \cdot \mathcal{F}(D_p)
\]

We discuss and show the inverse problem of circulant matrices below briefly according to [9-11]. Complex or real circulant matrices can be diagonalized in terms of a Fourier matrix which is subject to FFT for calculating the inverse of it. The complete proofs can be found in [10], [11]. We denote the \(N\) permutation matrix with \(\Pi(=\Pi_N)\) and show \(H_p\) as

\[
H_p = \sum_{k=0}^{N-1} H_{pk} \Pi^k
\]

For the elements \(y = [H_{p0}, \cdots, H_{p(N-1)}]\) the circulant matrix \(H_c\) can be written as a polynomial relationship which equals to \(P_y(\Pi)\). The characteristic polynomial \(P_y\) can be written as

\[
P_y(x) = \sum_{k=0}^{N-1} H_{pk}x^k
\]

We can denote the Fourier transformation’s primitive \(n\)-th root of unity as \(\omega := e^{-\frac{2\pi i}{n}}\). Using \(\omega\) the diagonal matrix \(\Omega(=\Omega_N)\) can be denoted as

\[
\Omega := \text{diag}(1, \omega, \omega^2, \cdots, \omega^{N-1})
\]

And finally the Fourier matrix \(F(=F_N)\) and its Hermitian adjoint \(F^*\) can be constructed based on \(\Omega(=\Omega_N)\). Now we can show the characteristic polynomial and \(H_c\) in terms of Fourier representation

\[
H_c(y) = F^* P_y(\Omega)F
\]

We can see that the Eigenvalues of \(H_c(y)\) will be \(P_y(\omega^k)\) which lets us invert \(H_c\) as suggested in \(O(N \cdot \log N)\) operations. These operations which are in frequency domain have to be performed for every path. For the example at the end of this paper and in our simulations we used the computations introduced in section (III-A).

C. Computational Complexity

We can group the calculations into two parts. The first group is the calculation of \(H\), Section (III-A). Here, we apply the FFT to (12) with \(O(N \cdot \log N)\) operations.

Apart from the calculations of \(H\) there are some additional computations, e.g. calculation of Doppler shifts and the addition operations which we can add to the second group of calculations. They have a linear computational complexity.

It is worth to note that the complexity of the best technique to benefit from Doppler diversity besides equalization was \(O(N^3)\) up to now [6]. In this work we propose a solution based on the FFT.

D. Channel Estimation

For this paper we assumed ideal channel knowledge to validate the equalizer and to separate its operation from the channel estimation. There are some recent effective channel estimation techniques for OFDM systems, e.g., basic matching pursuit (MP), orthogonal MP and recursive MP algorithms [7].

In these methods the channel is assumed to be sparse and the time-Doppler taps are identified sequentially by choosing the dominant component that will decrease the error function. In the future one of our goals will be to find a channel estimation algorithm which best meets the requirements of the proposed equalizer.

IV. SIMULATION AND RESULTS

In this section we demonstrate the performance of the proposed equalizer and compensation technique according to (III-A). Prior to the introduction and discussion of the results in a more complex scenario we describe the simulation process for simple two paths model with angles 0° and 45°, carrier frequency 862 MHz, speed 34.8 m/s and delays 0 and 1.6 μs. Here, angles mean that there is only one moving object seeing paths under those angles. For simplicity we take \(N = 4\) and \(X = [-1,1,1,1]\). After applying the IFFT we get \(x = [0,0,-1,0]\) which enters the LTV channel. In the LTV channel we have two paths which each can be denoted with \(h_1\) and \(h_2\) arrays after adding delays, Doppler shifts and attenuations. \(h_1\) and \(h_2\) arrays represent the channel impulse response for every symbol \(n = 1,\cdots,N\) in the channel according to (6) and can be written for our example after adding the delays as

\[
h_1 = [0,0,0.5,0]
\]

\[
h_2 = [-0.0000 + 0.0028i, -0.0028 - 0.0028i, 0.5 + 0.0028i, 0.0028 - 0.0028i]
\]
Now we can add Doppler shifts to the paths. Doppler shift arrays can be constructed using (8) and written as

\[ D_1 = [0.9901 + 0.1403i, 0.9606 + 0.2778i, 0.9122 + 0.4098i, 0.8457 + 0.5337i] \]

\[ D_2 = [0.9951 + 0.0994i, 0.9803 + 0.1977i, 0.9558 + 0.2941i, 0.9218 + 0.3877i] \]

After adding Doppler shifts to paths \( h_1 \) and \( h_2 \) we can sum and write them as

\[ h = \{-0.0003 + 0.0028i, -0.0022 - 0.0033i, 0.9331 + 0.3546i, 0.0037 - 0.0015i\} \]

In the LTV channel the received signal will be the convolution of the channel \( h \) and \( x \) and can be written as

\[ y = [0.0003 - 0.0028i, 0.0022 + 0.0033i, -0.9331 - 0.3546i, -0.0037 + 0.0015i]\]

This enters into the FFT block where we get

\[ Y = [-0.9343 - 0.3526i, 0.9352 + 0.3460i, -0.9313 - 0.3622i, 0.9316 + 0.3577i] \]

Finally we can equalize the \( Y \) signal by calculating the inverse of \( H \) which can be written as follows

\[ H = H_1 \otimes \mathcal{F}(D_1) + H_2 \otimes \mathcal{F}(D_2) \]

We can calculate \( H \) by applying FFT to (12) and write

\[ H_1 \otimes \mathcal{F}(D_1) = [0.4561 + 0.2049i, 0.4561 + 0.2049i, 0.4561 + 0.2049i, 0.4561 + 0.2049i]\]

\[ H_2 \otimes \mathcal{F}(D_2) = [0.4783 + 0.1477i, 0.4791 + 0.1411i, 0.4752 + 0.1573i, 0.4755 + 0.1528i]\]

The sum of them will be

\[ H = [0.9343 + 0.3526i, 0.9352 + 0.3460i, 0.9313 + 0.3622i, 0.9316 + 0.3577i] \]

The compensated signal will be \( \hat{Y} = Y/H = [-1, 1, -1, 1] \) which is equal to \( X \).

For simulations in more complex, realistic scenarios we implemented our approach in the DVB-T2 Common Simulation Platform (CSP) [12]. We assume the synchronization to be perfect. Additionally, we use the ideal channel estimation parameters except for the Doppler shift. We choose 8K 256-QAM SISO mode with guard interval \( \frac{1}{8} \). As a channel model we implement typical urban (TU) model with six paths, Table I.

Notice that even in the highest available channel (862 MHz) a Doppler shift of 200 Hz allows for a speed of 250 km/h.

Fig.(2) shows the inner decoder output bit error rate (BER) vs. angle of arrival estimation error (in degrees). The red curve demonstrates the performance of the original equalizer. Blue and green curves show the proposed equalizer's performance with given angle error. Fig.(3) demonstrates the performance under speed estimation errors (denoted in m/s). As we see the equalizer is robust to potentially inaccurate Doppler shift estimation. Both curves demonstrate the results in a noiseless channel.

V. CONCLUSION

We proposed a Doppler shift compensation technique and an equalizer which can compensate the channel distortions in time-variant scenarios with low complexity based on FFT. This technique compensates the Doppler shift in multipath propagation channels. Since the OFDM symbol duration typically is chosen smaller than the coherence time we assume that the Doppler shift is constant within an OFDM block. In our future work we focus on finding a channel estimator optimally fitted to the equalizer introduced in this paper. In addition we want to look into solutions for non-uniform Doppler effects within OFDM symbols.

REFERENCES


